Creep of current-driven domain-wall lines: intrinsic versus extrinsic pinning

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We present a model for current-driven motion of a magnetic domain-wall line, in which the dynamics of the domain wall is equivalent to that of an overdamped vortex line in an anisotropic pinning potential. This potential has both extrinsic contributions due to, e.g., sample inhomogeneities, and an intrinsic contribution due to magnetic anisotropy. We obtain results for the domain-wall velocity as a function of current for various regimes of pinning. In particular, we find that the exponent characterizing the creep regime depends strongly on the presence of a dissipative spin transfer torque. We discuss our results in the light of recent experiments on current-driven domain-wall creep in ferromagnetic semiconductors, and suggest further experiments to corroborate our model.

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I. INTRODUCTION

The driven motion of line defects through a disordered potential landscape has attracted considerable attention, for example in the context of vortices in superconductors,[1] wetting phenomena,[2], crack fronts,[3] and domain walls in ferromagnets.[4,5] The competition and interplay among the elasticity of the line, the pinning forces due to the disorder potential, and thermal fluctuations, lead to a wealth of physical phenomena. Topics discussed are, for example, the universality class of the roughening of the line, the nature of the pinning-depinning transition at zero temperature,[6] and the slide, depinning, and creep regimes of motion of the line that occur for decreasing driving field.[1,7]

The creep regime has been observed experimentally with field-driven motion of domain walls in ferromagnets.[4,5] This low-field regime is characterized by a nonlinear dependence of the domain-wall drift velocity $\langle \dot{X} \rangle$ on the external magnetic field $H_{\rm ext}$, given by

$$\langle \dot{X} \rangle \propto \exp\left\{ -\frac{E_c}{k_B T}\left(\frac{H_c}{H_{\rm ext}}\right)^{\mu_f}\right\}, \qquad (1)$$

where E_c is a characteristic energy scale, and H_c a critical field. The thermal energy is denoted by $k_B T$ and the exponent $\mu_f = (2\zeta - 1)/(2 - \zeta)$ is given in terms of the equilibrium wandering exponent ζ of the static line.[1,4,7] The phenomenological creep formula [Eq. (1)] is an Arrhenius law in which the energy barrier diverges for vanishing driving field, i.e., is "glassy". The underlying assumption is that there is a characteristic length scale that determines the displacement of the domain wall line. The validity of Eq. (1) has been confirmed both numerically[8] and with functional renormalization group methods.[9] It turns out that Eq. (1) is also valid in situations where roughening plays no role. For example, for a d-dimensional manifold driven through a periodic potential in $d + 1$ dimensions we have $\mu_f = d - 1$ (for $d \geq 2$).[1] Moreover, in the regime where the line defect moves via variable-range hopping, we have that $\mu_f = 1/3$ if the motion is in two dimensions.[1,10,11]

In addition to magnetic-field driven motion, a lot of recent theoretical and experimental research has been devoted to manipulating domain walls with electric current[12,13,14,15,16,17,18,19,20,21] via so-called spin transfer torques.[22,23,24,25] Domain wall motion driven by a current is quite distinct from the field-driven case. For example, it has been theoretically predicted that, in certain regimes of parameters, the domain wall is intrinsically pinned at zero temperature, meaning that there exists a nonzero critical current even in the absence of disorder.[12] In clean samples, the phenomenology of current-driven domain wall motion turns out to crucially depend on the ratio of the dissipative spin transfer torque parameter[26] β and the Gilbert damping constant α_G.[13] Although theoretical predictions[27,28,29,30] indicate that, at least for model systems, this ratio differs from one, it turns out to be difficult to extract its precise value from experiments on current-driven domain wall motion, to a large extent because disorder and nonzero-temperature effects[21,31] complicate theoretical calculations of the domain wall drift velocity for a given current. This is the first motivation for the work presented in this paper.

Previous work on current-driven domain wall motion at nonzero temperature focused on *rigid* domain walls. Tatara *et al.*[32] found that $\ln\langle \dot{X} \rangle$ was proportional to the current density j. Discrepancy of this result with experiments,[21] that did not observe linear dependence of wall velocity on current, motivated the more systematic inclusion of nonzero-temperature effects on rigid-domain wall motion by Duine *et al.*,[31] who found that $\ln\langle \dot{X} \rangle \propto \sqrt{j}$ in certain regimes. Although the latter was an important step in qualitatively understanding the experimental results of Yamanouchi *et al.*,[21,33] a detailed understanding of these experiments is still lacking and this is the second motivation of this paper. For completeness, we mention also the theoretical work by Martinez *et al.*[34,35] who considered thermally-assisted current-driven rigid domain wall motion in the regime of

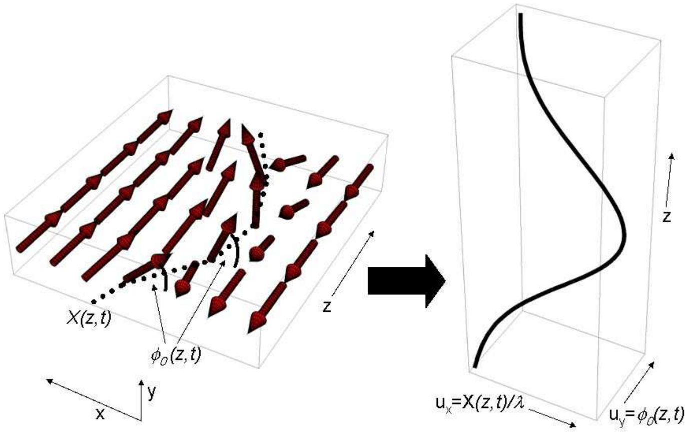

Fig. 1: Duine/Morais Smith

FIG. 1: Mapping of current-driven domain wall dynamics to that of a vortex line. The position of the domain wall $X(z,t)$ and its chirality $\phi_0(z,t)$ become the position (u_x, u_y) of the vortex via $(u_x, u_y) \equiv (X/\lambda, \phi_0)$. The potential landscape for this vortex is in general anisotropic. In particular, the tilting in the u_x-direction is set by the external magnetic field and the dissipative spin transfer torque. The tilting in the u_y-direction is determined by the reactive spin transfer torque.

large anisotropy, where the chirality of the domain wall plays no role and the pinning is essentially dominated by extrinsic effects. Furthermore, Ravelosona *et al.*[36] have observed thermally-assisted domain wall depinning, and Laufenberg *et al.*[37] have determined the temperature dependence of the critical current for depinning the domain wall.

In this paper we present a model for a current-driven *elastic* domain-wall line moving in *one dimension* in the presence of *disorder* and *thermal fluctuations*. A crucial ingredient in the description of current-driven motion is the chirality of the domain wall, which acts like an extra degree of freedom. This enables a reformulation of current-driven domain wall motion as a vortex line moving in an anisotropic potential in *two dimensions* (see Fig. 1), which we present in detail in Sec. II. Using this physical picture, we analyze in Sec. III the different regimes of pinning within the framework of collective pinning theory.[1] We present results on the velocity of the domain-wall line as a function of current, both in the regime where intrinsic pinning due to magnetic anisotropy dominates, and in the extrinsic-pinning-dominated regime. Finally, we discuss in Sec. IV our theoretical results in relation to recent experiments on current-driven domain walls in GaMnAs.[33] Although these experiments remain, in our opinion, not fully understood, we suggest that they may be explained by assuming a specific form of the pinning potential for the domain-wall line. We suggest futher experiments that could corroborate this suggestion.

II. DOMAIN WALL AS A VORTEX LINE

The equation of motion for the magnetization direction $\mathbf{\Omega}$ in the presence of a transport current j is, to lowest order in temporal and spatial derivatives, given by

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_{\rm s} \cdot \nabla\right) \mathbf{\Omega} - \mathbf{\Omega} \times (\mathbf{H} + \mathbf{H}_{\rm ext} + \boldsymbol{\eta}) = -\alpha_G \mathbf{\Omega} \times \left(\frac{\partial}{\partial t} + \frac{\beta}{\alpha_G} \mathbf{v}_{\rm s} \cdot \nabla\right) \mathbf{\Omega}\ . \quad (2)$$

The left-hand side of this equation contains the reactive[26] spin transfer torque[38] proportional to the velocity $\mathbf{v}_{\rm s} = Pj/(e\rho_{\rm s})$. The latter velocity characterizes the efficiency of spin transfer. Here, P is the polarization of the current in the ferromagnet, e is the carrier charge, and the spin density is denoted by $\rho_{\rm s} \equiv 2/a^3$ with a the lattice constant. The other terms on the left-hand side of Eq. (2) describe precession around the external field $\mathbf{H}_{\rm ext}$ and the effective field $\mathbf{H} = -\delta E[\mathbf{\Omega}]/(\hbar\delta\mathbf{\Omega})$, which is given by a functional derivative of the energy functional $E[\mathbf{\Omega}]$ with respect to the magnetization direction. The stochastic magnetic field $\boldsymbol{\eta}$ incorporates thermal fluctuations, and it has zero mean and correlations determined by the fluctuation-dissipation theorem[39]

$$\langle \eta_\sigma(\mathbf{x}, t)\eta_{\sigma'}(\mathbf{x}', t')\rangle = \frac{2\alpha_G k_{\rm B} T}{\hbar}\delta(t - t')a^3\delta(\mathbf{x} - \mathbf{x}')\delta_{\sigma\sigma'}\ . \quad (3)$$

It can be shown that the above still holds in the presence of current, at least to first order in the applied electric field[30] that drives the transport current. The fluctuation-dissipation theorem also ensures that in equilibrium the probability distribution for the magnetization direction is given by the Boltzmann distribution $\mathcal{P}[\mathbf{\Omega}] \propto \exp\{-E[\mathbf{\Omega}]/k_{\rm B}T\}$. The right-hand side of Eq. (2) contains only dissipative terms. The Gilbert damping term is proportional to the damping parameter α_G, and the dissipative[26] spin transfer torque is characterized by the dimensionless parameter β.[13]

We consider a ferromagnet with magnetization direction $\mathbf{\Omega} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ that depends only on the the x and z-direction. In addition, we take the current in the x-direction and the external magnetic field in the z-direction. The size of the ferromagnetic film in the α direction is denoted by L_α ($\alpha \in \{x, y, z\}$) and we assume that $L_y \ll L_z$. The latter assumption allows us to model the domain wall as a line. Furthermore, we take the ferromagnet to have an easy z-axis and hard y-axis, with anisotropy constants K and $K_\perp$, respectively. The spin stiffness is denoted by J. With these assumptions, static domain walls have a width $\lambda = \sqrt{J/K}$, and are, for the simplest model to be discussed in more detail below [see Eq. (10)], described by the solutions $\theta_0(\mathbf{x}) = \cos^{-1}[\tanh(x/\lambda)]$ and $\phi(\mathbf{x}) = 0$. To arrive at a description of the dynamics of the domain wall, we use two collective coordinates which may depend on the z-coordinate, so that the domain wall is modelled as a line. The collective coordinates are the position of the

wall $X(z,t)$ and the chirality $\phi_0(z,t)$. The latter determines the sense in which the magnetization rotates upon going through the domain wall. The result of Ref. [31] is straightforwardly generalized to the case of a domain wall line. This amounts to solving Eq. (2) variationally with the *ansatz* $\theta_{\rm dw}(\mathbf{x},t) = \theta_0\left((x - X(z,t))/\lambda\right)$ and $\phi_{\rm dw}(\mathbf{x},t) = \phi_0(z,t)$, which yields the equations of motion

$$\begin{aligned}
\frac{\partial \phi_0}{\partial t} + \frac{\alpha_G}{\lambda}\frac{\partial X}{\partial t} &= \frac{-a^3}{2\hbar L_y \lambda}\frac{\delta V}{\delta X} + \frac{\beta v_{\rm s}}{\lambda} - H_{\rm ext} + \eta_X(z,t) \ ; \\
\frac{1}{\lambda}\frac{\partial X}{\partial t} - \alpha_G \frac{\partial \phi_0}{\partial t} &= \frac{a^3}{2\hbar L_y \lambda^2}\frac{\delta V}{\delta \phi_0} + \frac{v_{\rm s}}{\lambda} + \eta_\phi(z,t) \ ,
\end{aligned} \tag{4}$$

where the domain-wall energy

$$V[X,\phi_0] \equiv E[\theta_{\rm dw},\phi_{\rm dw}] \ , \tag{5}$$

and the stochastic forces are determined from

$$\begin{aligned}
&\langle \eta_\phi(z,t)\eta_\phi(z',t')\rangle = \langle \eta_X(z,t)\eta_X(z',t')\rangle \\
&= \left(\frac{\alpha_G k_{\rm B} T}{\hbar}\right)\left(\frac{a^3}{\lambda^2 L_y}\right)\delta\left(\frac{z-z'}{\lambda}\right)\delta(t-t') \ .
\end{aligned} \tag{6}$$

The above equations are derived using a variational method for stochastic differential equations based on their path-integral formulation.[31,40] Their validity is confirmed *a posteriori* by noting that in equilibrium the probability distribution function for the position and chirality of the domain wall is the Boltzmann distribution. That is, the Fokker-Planck equation for the probability distribution $\mathcal{P}[X,\phi_0]$ of the domain wall position and chirality that follows from Eqs. (4) and (6), is given by[41]

$$\begin{aligned}
&\left(1+\alpha_G^2\right)\frac{2\hbar L_y \lambda^2}{a^3}\frac{\partial \mathcal{P}[X,\phi_0]}{\partial t} = \\
&\int dz \frac{\delta}{\delta X(z)}\left(\alpha_G \lambda \frac{\delta V}{\delta X(z)} - \frac{\delta V}{\delta \phi_0(z)}\right)\mathcal{P}[X,\phi_0] \\
&+\int \frac{dz}{\lambda}\frac{\delta}{\delta \phi_0(z)}\left(\lambda \frac{\delta V}{\delta X(z)} + \alpha_G \frac{\delta V}{\delta \phi_0(z)}\right)\mathcal{P}[X,\phi_0] \\
&+\alpha_G k_B T \int \frac{dz}{\lambda}\left(\frac{\delta^2}{\delta \phi_0^2(z)} + \lambda^2 \frac{\delta^2}{\delta X^2(z)}\right)\mathcal{P}[X,\phi_0]
\end{aligned} \tag{7}$$

Upon insertion of the Boltzmann distribution $\mathcal{P}_{\rm eq}[X,\phi_0] \propto \exp\{-V[X,\phi_0]/(k_{\rm B}T)\}$ into this equation, one straightforwardly verifies that it is indeed a stationary solution.

Rewriting the equations of motion for the domain wall position and chirality in terms of the dimensionless coordinate $\mathbf{u}(z,t) \equiv (X(z,t)/\lambda, \phi_0(z,t))$, we find from Eq. (4) that the domain wall is described by

$$\epsilon_{\alpha\alpha'}\dot{u}_{\alpha'}(z,t) = -\alpha_G \dot{u}_\alpha(z,t) - \frac{\delta \tilde{V}[\mathbf{u}]}{\delta u_\alpha(z,t)} + \eta_\alpha(z,t) \ , \tag{8}$$

with $\epsilon_{\alpha\alpha'}$ the two-dimensional Levi-Civita symbol. (Summation over repeated indices $\alpha, \alpha' \in x, y$ is implied. Note that $\eta_\alpha = \eta_{X,\phi}$ for $\alpha = x, y$.) The above equation of motion [Eq. (8)] corresponds to the overdamped limit of vortex-line dynamics in an anistropic potential $\tilde{V}[\mathbf{u}]$. The left-hand side of Eq. (8) corresponds to the Magnus force on the vortex. We emphasize that a mass term is missing, indicating we are indeed dealing with the overdamped limit of vortex motion. (Note that the mass of the fictitious vortex is not related to the Döring domain wall mass[42] that arises from eliminating the chirality from the domain-wall description, which is valid provided the latter is small.[43] As the dynamics of the domain-wall chirality is essential for current-driven domain wall motion, this latter approximation is not sufficient for our purposes.) The right-hand side of the equation of motion contains a damping term proportional to α_G, and a term representing thermal fluctuations. The force is determined by the potential

$$\tilde{V} = \frac{a^3 V[\lambda u_x, u_y]}{2\hbar L_y \lambda^2} + \int \frac{dz}{\lambda}\left[\left(\frac{\beta v_{\rm s}}{\lambda} - H_{\rm ext}\right)u_x + \frac{v_{\rm s}}{\lambda}u_y\right] \ . \tag{9}$$

The tilting of this potential in the u_x-direction is determined by the parameter β, the current $v_{\rm s}$, and the external field $H_{\rm ext}$. The tilting in the u_y-direction is determined only by the current. The model in Eqs. (8) and (9), illustrated in Fig. 1, is the central result of this paper. In the following section, we obtain results from this model for the domain wall velocity in different regimes of pinning, specializing to the case of current-driven domain wall motion ($H_{\rm ext} = 0$).

III. DOMAIN WALL CREEP

In this section, we obtain results for the average drift velocity of the domain wall as a function of applied current. First, we discuss the situation without disorder, hereafter we incorporate the effects of disorder.

A. Intrinsic pinning

In this subsection, we make two assumptions that are not related. First, we consider a homogeneous system, i.e., a system without disorder potential $V_{\rm pin} = 0$. Second, we take $\beta = 0$. In this case, the domain wall is intrinsically pinned.[12] This comes about as follows. For the magnetic nanowire model discussed in the previous section, the energy functional in the clean limit is given by

$$\begin{aligned}
E[\mathbf{\Omega}] = \int \frac{d\mathbf{x}}{a^3}&\left\{\frac{J}{2}\left[(\nabla\theta)^2 + \sin^2\theta\,(\nabla\phi)^2\right]\right. \\
&\left. + \frac{K}{2}\sin^2\theta + \frac{K_\perp}{2}\sin^2\theta\sin^2\phi\right\} \ .
\end{aligned} \tag{10}$$

Upon insertion of the domain wall *ansatz* into the above energy functional, we find that

$$\tilde{V}[\mathbf{u}] = \int \frac{dz}{\lambda}\left[\frac{J}{2\hbar}\left(\frac{\partial \mathbf{u}}{\partial z}\right)^2 - \frac{K_\perp}{4\hbar}\cos(2u_y) + \frac{v_{\rm s}}{\lambda}u_y\right] . \tag{11}$$

Because the above potential is independent of z, the domain wall remains straight at zero temperature, i.e., $\partial\mathbf{u}/\partial z = 0$. By solving the equations of motion in Eq. (8) for the potential in Eq. (11) at zero temperature and for a straight domain wall, one finds that $\langle|\dot{\mathbf{u}}|\rangle \propto \sqrt{v_{\rm s}^2 - (\lambda K_\perp/2\hbar)^2}$ so that the domain wall is pinned up to a critical current given by $j_c = \lambda K_\perp e\rho_{\rm s}/2\hbar P$. (The brackets $\langle\cdots\rangle$ denote time and thermal average.) This intrinsic pinning is entirely due to the anisotropy energy,[12] determined by $K_\perp$, and does not occur for field-driven domain-wall motion, or current-driven domain-wall motion with $\beta \neq 0$. Physically, it comes about because, for the model of a domain wall that we consider here, the reactive spin transfer torque causes the magnetization to rotate in the easy plane. This corresponds to an effective field that points along the hard axis. Because the Gilbert damping causes the magnetization to precess towards the effective field, the current tilts the magnetization out of the easy plane. This leads to a cost in anisotropy energy which stops the drift motion of the domain wall if the current is too small. By solving the equations of motion for the potential in Eq. (11) at nonzero temperature in the limit of a straight wall, one recovers the result of Ref. [31].

At nonzero temperature, the domain wall is no longer straight. Since only the chirality is important, our model for current-driven domain-wall motion in Eq. (11) then corresponds to the problem of a string in a tilted-washboard potential, that has been studied before[44] in different contexts. At nonzero temperature the string propagates through the tilted-washboard potential by nucleating a kink-antikink pair in the z-direction of the domain-wall chirality $\phi_0(z,t)$. The kink and antikink are subsequently driven apart by the tilting of the potential, which results in the propagation of the string.

In the limit when the current is close to the critical one, a typical energy barrier is determined by the competition between the elasticity of the string and the tilted potential.[1] For $(j_c - j)/j_c \ll 1$ the cosine in the energy functional in Eq. (11) may be expanded around one of its minima, which yields

$$\tilde{V}[\mathbf{u}] = \int \frac{dz}{\lambda}\left[\frac{J}{2\hbar}\left(\frac{\partial \delta u_y}{\partial z}\right)^2 + \frac{K_\perp}{\hbar}\sqrt{1-\left(\frac{j}{j_c}\right)^2}\,\delta u_y^2 + \frac{2v_s}{3\lambda}\delta u_y^3\right] , \tag{12}$$

where we have omitted an irrelevant constant. In the above expression, δu_y denotes the displacement from the minimum. Note that we have dropped the dependence of the potential on u_x which is allowed because the potential is not tilted in the u_x-direction (provided that $\beta = 0$).

The potential in Eq. (12) has a minimum for $\delta u_y^{\rm min} = 0$ (by construction) and a maximum for $\delta u_y^{\rm max} = -v_s K_\perp\sqrt{1-(j/j_c)^2}/\lambda\hbar$. The pinning potential energy barrier, i.e., the pinning potential evaluated at the maximum, scales as $\Delta V \propto [1-(j/j_c)^2]^{3/2}$. Consider now the situation that a segment of length L of the string is displaced from the minimum and pinned by the maximum of the potential. The length L is then determined by the competition between the elastic energy $\sim J(\delta u_y^{\rm max}/L)^2$, that tends to keep the domain wall straight, and the pinning potential ΔV. Equating these contributions yields for the length L that

$$L \propto \left[1-\left(\frac{j}{j_c}\right)^2\right]^{-1/4} . \tag{13}$$

The typical energy barrier that thermal fluctuations have to overcome to propagate the domain wall is then given by evaluating Eq. (12) for a segment of this length. This yields a typical energy barrier $\propto [1-(j/j_c)^2]^{5/4}$. Putting these results together and assuming an Arrhenius law, we find that the domain wall velocity is

$$\ln\langle|\dot{\mathbf{u}}|\rangle \propto -\frac{1}{k_BT}\frac{JL_y}{a^3}\sqrt{\frac{K_\perp}{K}}\left[1-\left(\frac{j}{j_c}\right)^2\right]^{5/4} , \tag{14}$$

for $(j_c - j)/j_c \ll 1$.

In the regime of small currents $j \ll j_c$, the typical energy barrier depends only logarithmically on the current,[44] so that $\langle|\dot{\mathbf{u}}|\rangle \propto j$. This latter result for the domain-wall velocity is the same as found from a treatment of rigid domain-wall motion at nonzero temperature[31] in the limit that $j \ll j_c$. This is understood by noting that in the limit of vanishing current the elasticity of the domain-wall line does not enter the expression for the typical energy barrier,[44] the so-called "thin-wall" limit.[45]

B. Extrinsic pinning

We now add extrinsic pinning, i.e., a disorder potential $V_{\rm pin}$ to the potential in Eq. (11). Following Ref. [12] we assume, in first instance, that it only couples to the position of the domain wall u_x and not to its chirality u_y. This assumption is made mainly to simplify the problem. Considering now the general case that also $\beta \neq 0$, we have that

$$\tilde{V}[\mathbf{u}] = \int \frac{dz}{\lambda}\left[\frac{J}{2\hbar}\left(\frac{\partial \mathbf{u}}{\partial z}\right)^2 - \frac{K_\perp}{4\hbar}\cos 2u_y + V_{\rm pin}(u_x,z) + \beta\frac{v_{\rm s}}{\lambda}u_x + \frac{v_{\rm s}}{\lambda}u_y\right] . \tag{15}$$

We estimate a typical energy barrier using the collective pinning theory.[1,7] Therefore, we assume that we are in the regime where the pinning energy grows sublinearly with the length of the wall, and that there exists a typical length scale L at which domain-wall motion occurs. (Note that we take L dimensionless since the coordinate $\mathbf{u}$ is dimensionless.)[1] The energy of a segment of this length that is displaced is given by

$$E(L) = \epsilon_{\rm el}\frac{u_x^2}{L} + \beta\frac{v_{\rm s}}{\lambda}Lu_x + \frac{v_{\rm s}}{\lambda}Lu_y \ . \qquad (16)$$

The first term is the elastic energy with $\epsilon_{\rm el} = J/2\hbar\lambda^2$. The second and third term correspond to the dissipative and reactive spin transfer torques, respectively. Note that since the dissipative spin transfer torque acts like an external magnetic field we are able to incorporate it in the above energy. The potential $V_{\rm pin}(u_x, z)$ leads to roughening in the u_x direction. Following standard practice,[1,4,7] we assume a scaling law $u_x(L) = u_{x0}L^\zeta$, with ζ the equilibrium wandering exponent, already mentioned in the introduction, and u_{x0} a constant. The displacement in the u_y direction is not roughened, because we have assumed that $V_{\rm pin}(u_x, z)$ does not depend on u_y, i.e., the domain-wall chirality. Rather, the displacement in this direction is determined by the minima of the potential in Eq. (11) and we have that $u_y = u_{y0}$ independent of L for $j \ll j_c$. Note that in this limit the elastic energy due to displacement in the u_y-direction can also be neglected.[1] Hence, we find that

$$E(L) = \epsilon_{\rm el}u_{x0}^2L^{2\zeta-1} + \beta\frac{v_{\rm s}}{\lambda}u_{x0}L^{\zeta+1} + \frac{v_{\rm s}}{\lambda}Lu_{y0} \ . \qquad (17)$$

Minimizing this expression with respect to L then leads to a typical energy barrier. Assuming an Arrhenius law,[1,4,7] we find for the domain wall velocity

$$\ln\langle|\dot{\mathbf{u}}|\rangle \propto -\frac{\epsilon_{\rm el}}{k_BT}\left(\frac{j_c}{j}\right)^{\mu_c} \ . \qquad (18)$$

For $\beta = 0$ we have that $\mu_c = (2\zeta-1)/(2-2\zeta)$. For $\beta \neq 0$ we find $\mu_c = (2\zeta-1)/(2-\zeta)$. In particular, for $\zeta = 2/3$, applicable to domain walls in ferromagnetic metals,[4] we have $\mu_c = 1/2$ for $\beta = 0$, and $\mu_c = 1/4$ for $\beta \neq 0$. Since the dissipative spin transfer torque, proportional to β, acts like an external magnetic field on the domain wall [see Eqs. (8)], we recover the usual results for field-driven domain wall motion[4] from our model. This result is also understood from the fact that an external magnetic field does not tilt the domain wall potential in the chirality direction, as opposed to a current, so that the domain wall chirality plays no role in field-driven domain wall creep. We note that Eq. (4), or, equivalently, Eq. (8), contains a description of Walker breakdown[46] in the clean zero-temperature limit and is also able to describe the transition from the creep regime to the regime of precessional field-driven domain-wall motion observed recently.[47]

IV. DISCUSSION AND CONCLUSIONS

In very recent experiments on domain walls in the ferromagnetic semiconductor GaMnAs, Yamanouchi *et al.*,[33] have observed field-driven domain-wall creep with exponent $\mu_f \simeq 1$, and current-driven creep with $\mu_c \simeq 1/3$, over 5 orders of magnitude of domain-wall velocities. The fact that these two exponents are different could imply that β is extremely small for this material. For $\beta = 0$ and the specific pinning potential discussed in the previous section it is, however, impossible to find a single roughness exponent that yields both $\mu_f = 1$ and $\mu_c = 1/3$. (Note that the theoretical arguments in Ref. [33] give $\mu_f = 1$ and $\mu_c = 1/2$.)

Although it is extremely hard to determine the microscopic features of the pinning potential, we emphasize that if pinning is not provided mainly by *point-like* defects (as considered in this paper and argued by Yamanouchi *et al.* to be the case in their experiments,[33]) but consists of random *extended* defects, the creep exponents would change dramatically. Indeed, the latter type of disorder, which could occur in samples if there are, e.g., steps in the height of the film, allows for a variable-range hopping regime for creep in which the exponent $\mu = 1/3$ in the two-dimensional case. Moreover, upon increasing the driving force, a crossover occurs to the so-called half-loop regime where the exponent $\mu = 1$.[1,10] An alternative explanation for the experimental results of Yamanouchi *et al.*[33] would be that $\beta \neq 0$ so that the behavior for field and current-driven motion is similar. If the pinning potential is random and extended, it would be possible that the current-driven experiment is probing the variable-range hopping regime with $\mu = 1/3$, whereas the field-driven case probes the half-loop regime with $\mu = 1$. This scenario would also reconcile the results of Ref. [33] with previous ones[21] which yielded a critical exponent of $\mu \simeq 0.5$, as the latter could be in a different regime of pinning. In conclusion, further experiments are required to clarify this issue. The conjecture of pinning by extended defects may be experimentally verified by increasing the driving in the current-driven case and checking if the exponent crosses over from $\mu = 1/3$ to $\mu = 1$, while remaining in the creep regime. Finally, since the exponent $\mu = 1/3$ occurs strictly for variable-range hopping in two dimensions, we note that the mapping presented in this paper is crucial in obtaining this result.

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